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On the Possibility of Credit Rationing in the Stiglitz-Weiss Model

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Abstract

Contrary to what is consistently assumed in the literature, the return function cannot be hump-shaped in the Stiglitz-Weiss (1981) model. This has important consequences for the possible occurrence of credit rationing and redlining. With a single class of borrowers, banks offer credit in two stages. Demand possibly exceeds supply in stage one, but not in stage two. With several observationally distinguishable borrower classes, the firms in a borrower class are redlined only under circumstances which imply that they would not get credit in a perfect capital market either.

JEL classification: D82, E51, G21

Key words: credit rationing, redlining, asymmetric information.

In one of the ground-breaking papers in financial economics, Joseph Stiglitz and Andrew Weiss (1981) analyze several models of “Credit Rationing in Markets with Imperfect Information”. When Stiglitz was awarded the Nobel Prize in Economics in 2001, together with George Akerlof and Michael Spence, this article was mentioned implicitly in the press release and explicitly in the advanced information (see http://nobelprize.org/economics/laureates/2001/press.html). The most prominent of these models, used in many subsequent research papers and reproduced in many textbooks and survey articles, is the one with a continuum of borrower types endowed with projects with identical expected internal rates of return and with non-observable riskiness. In this paper, we re-examine this model. Stiglitz and Weiss (1981, pp. 394, 397) and the contributors to the strand of the literature originating from their seminal work derive several propositions using the assumption that the return function (i.e., the relation between the interest rate charged and the resulting expected rate of return on lending) is hump-shaped, with a unique interior maximum. Our point of departure is the observation that, actually, the return function cannot be hump-shaped. It is not necessarily monotonic, but it attains its global maximum at the maximum interest rate beyond which there is no demand for credit. In view of this, it is curious to see how the hump shape has been making its way into articles and textbooks ever since the publication of Stiglitz and Weiss (1981). Even so, this flaw would not be very remarkable if the hump shape assumption served expositional convenience only, while the substance of the analysis were unaffected. We proceed to show that this is not so. With a single class of borrowers, equilibrium credit rationing can occur with a hump-shaped return function. Taking into account the fact that the return function is not hump-shaped, banks offer credit in two stages. In equilibrium, there may be excess demand in stage one, but not in stage two. The question of whether credit rationing then prevails becomes a semantic issue. Moreover, having introduced observationally distinguishable classes of firms to their model, Stiglitz and Weiss (1981, pp. 406-407) emphasize the possible prevalence of “redlining” (i.e., entire classes of firms being excluded from the credit market). We show that given that the classes’ return functions attain their global maxima at the respective maximum interest rates, a class can only be redlined if their projects’ expected internal rate of return falls short of the rate of return required by suppliers of capital. In this instance, they would not get credit in a perfect capital market either. It goes without saying that the aim of our re-examination is not to deny the crucial importance of asymmetric information in understanding how financial markets work. Rather, we intend to fill some still existing gaps in our understanding of how one of the most important models we use to analyze these markets works.

Section I briefly repeats the assumptions of the Stiglitz-Weiss (1981) model. The fact that the return function cannot be hump-shaped is proved in Section II. Section III introduces some technical assumptions needed to give a clean characterization of the credit market equilibrium in Section IV below.

The model with distinguishable borrower classes is the object of Section V. Section VI concludes.

I Model

In this section, we briefly recapitulate the assumptions of the Stiglitz-Weiss (1981) model. We do not modify or extend the model and employ the notation introduced by Stiglitz and Weiss (1981).
The model covers two time periods. There is a continuum of length N of firms of different types, θ. $G(\theta)$ is the distribution of firm types and has support $[0, \theta^{max}]$. In period 1, each firm has access to one indivisible investment project with uncertain payoff, R (≥ 0), in period 2. The distribution of returns to the projects of type-θ firms is denoted as $F(R|\theta)$. All types of projects have the same expected return, $\bar{R}$ (> 0): $E_R(R|\theta) = \bar{R}$. It is assumed that if $\theta' > \theta''$, the distribution $F(R|\theta')$ is a mean-preserving spread of $F(R|\theta'')$: $\int_0^x F(R|\theta')dR \geq \int_0^x F(R|\theta'')dR$ for all $x \geq 0$. In this sense, the higher θ, the riskier the projects. Each project requires a capital input, B $(0 < B < \bar{R})$. There is asymmetric information: firms observe their own type, θ, while other agents do not. Banks channel funds from lenders to firms. The banking sector does not use resources other than financial capital and is perfectly competitive. So the depositors' expected rate of return is equal to the expected rate of return on lending, ρ. The only financial instruments banks use are standard credit contracts. Because of non-observable borrower types, the interest rate(s) charged, r, cannot be made contingent on borrower types, θ. Each firm that receives credit posts an exogenous amount C $(0 < C < B)$ as collateral. (All results derived in this paper also hold true if C is used as internal finance, so that the amount borrowed is $B - C$.) Both firms and lenders are risk-neutral. As for banks, one can alternatively assume that the amount of funds lent by each single bank is so large that diversification eliminates risk. Firms apply for credit if the expected return on their investment project is non-negative. The supply of capital, $L^S(\rho)$, is assumed to be an increasing function of the expected rate of return, ρ ($L^S(\rho)$ can be thought of as the solution to a two-period utility maximization problem with given levels of income in period 1 and non-interest income in period 2).

II The return function cannot be hump-shaped

In this section, we show that the return function, which relates ρ to r, attains its unique global maximum at the maximum interest rate beyond which there is no demand for capital. The implications of this finding will be elaborated in subsequent sections.
Firms' profit is $\pi(R, r) = \max\{R - (1 + r)B, -C\}$. Riskier projects have a higher expected profit, $E_R[\pi(R, r)|\theta] = \bar{R} - (1 + r)B + \int_0^{(1+r)B-C} F(R|\theta)dR$. Firms apply for credit if $E_R[\pi(R, r)|\theta] \geq 0$.

This condition can be rewritten as $\theta \geq \theta(r)$, where $\theta(r)$ is defined by $E_R[\pi(R,r)|\theta(r)] \geq 0$ and $E_R[\pi(R,r)|\theta] < 0$ for $\theta < \theta(r)$ (i.e., $E_R[\pi(R,r)|\theta(r)] = 0$ if $F(R|\theta)$ and, hence, $E_R[\pi(R,r)|\theta]$ are continuous in θ, see Stiglitz and Weiss, 1981, Theorem 1, p. 396). This shows that there is adverse selection, in that only sufficiently risky projects are realized. Obviously, the critical value, $\theta(r)$, increases as r rises (see Stiglitz and Weiss, 1981, Theorem 2, p. 396). That is, increases in the interest rate aggravate the adverse selection problem. Or, put differently, as r rises, firms with a lower value of θ cease to demand loans earlier. Let r^{max} denote the interest rate at which the riskiest firms (those of type θ^{max}) cease to demand funds: $\theta(r^{max}) = \theta^{max}$. Interest rates $r > r^{max}$ are economically irrelevant because the demand for capital is zero. The repayment a bank receives is $\min\{(1+r)B, R+C\} = R - \pi(R,r)$. Since banks cannot distinguish between firms of different riskiness, θ, but know that only firms with $\theta \geq \theta(r)$ apply for loans, the expected repayment is $\bar{R} - E_{R,\theta}[\pi(R,r)|\theta \geq \theta(r)]$. Perfect competition in banking thus implies

$$\rho = \frac{\bar{R} - E_{R,\theta}[\pi(R,r)|\theta \geq \theta(r)]}{B} - 1. \tag{1}$$

This equation assigns a unique expected rate of return, ρ, to each interest rate, r, and thus determines the return function, $\rho(r)$. Clearly, $\rho(r)$ is not necessarily monotonically increasing. As r rises, the adverse selection effect ($\theta(r)$ rises) may outweigh the direct positive effect of higher contractual repayments (see Stiglitz and Weiss, 1981, p. 397).

Stiglitz and Weiss (1981, pp. 394, 397) used a hump-shaped return function in order to demonstrate the possible existence of equilibrium credit rationing. This hump-shaped return function has subsequently made its way into numerous articles and textbooks. Curiously, it has gone unnoticed for more than two decades that the return function cannot be hump-shaped (see the upper panel of Figure 1):

THEOREM 1: *The return function, $\rho(r)$, attains its unique global maximum at the maximum interest rate, r^{max}.*

Proof: As shown above, $E_R[\pi(R,r)|\theta] \geq 0$ for all $\theta \geq \theta(r)$. The inequality is strict for $\theta = \theta^{max}$ for all $r < r^{max}$. Therefore, $E_{R,\theta}[\pi(R,r)|\theta \geq \theta(r)] > 0$ and, from (1),

$$\rho < \frac{\bar{R}}{B} - 1, \text{ for all } r < r^{max}. \tag{2}$$

Now let $r \to r^{max}$ from below. As all but the riskiest group of investors cease to demand capital,

$$E_{R,\theta}[\pi(R,r)|\theta \geq \theta(r)] \to E_R[\pi(R,r)|\theta^{max}] \text{ as } r \to r^{max}.$$

Furthermore, by the definition of r^{max},

$$E_R[\pi(R,r)|\theta^{max}] \to 0 \text{ as } r \to r^{max}.$$

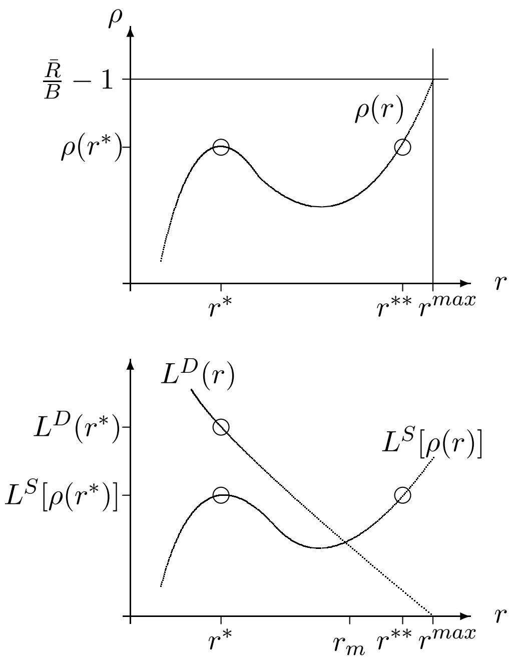

Figure 1: Return function and credit market equilibrium

Together with (1), it follows that

$$\rho \to \frac{\bar{R}}{B} - 1 \text{ as } r \to r^{max}. \tag{3}$$

Equations (2) and (3) prove that the return function attains a unique global maximum at $r = r^{max}$; it cannot be hump-shaped. Q.E.D.

The intuition for this result is straightforward. The return on investment is divided between borrowers and lenders. As long as there are borrowers who make a strictly positive expected profit, the return on lending, ρ, falls short of the investment projects' expected internal rate of return, $\bar{R}/B - 1$. Since whenever other firms are still active, firms in the highest risk group, θ^{max}, make a strictly positive expected profit, the only way to "squeeze" all the profits out of the borrowers is to set the interest rate at the level which makes the highest risk group, θ^{max}, just indifferent between realizing their projects or not.

III Demand and supply

In this section, we describe demand and supply in the credit market (see the lower panel of Figure 1). Other than usual in the literature, we specify the parameters and functions involved in such detail that in the subsequent sections we shall be able to give a complete characterization of the credit market equilibrium, rather than merely provide examples. Obviously, this level of rigor is indispensable if we want to rule out certain properties of credit market equilibria.

The demand for capital is $L^D(r) = NB\{1 - G[\theta(r)]\}$. The supply of funds is $L^S[\rho(r)]$. In order to abstract from existence problems, we make the following continuity and boundary assumptions.

ASSUMPTION 1: *$F(R|\theta)$, $G(\theta)$, and $L^S(\rho)$ are continuous.*

The continuity of $F(R|\theta)$ implies that $E_R[\pi(R,r)|\theta]$ and, hence, $\theta(r)$ are continuous. Together with the continuity of $G(\theta)$, it follows that $L^D(r)$ is continuous. Since $E_R[\pi(R,r)|\theta]$, $\theta(r)$, and $F(R|\theta)$ are continuous, so is $\rho(r)$. Together with the continuity of $L^S(\rho)$, it follows that the supply of loans, $L^S[\rho(r)]$, is continuous.

ASSUMPTION 2: *$L^S(0) \leq 0$ and $L^S(\bar{R}/B - 1) > 0$.*

We have $E_R[\pi(R,0)|\theta] = \bar{R} - B + \int_0^{B-C} F(R|\theta)dR > 0$. This implies that if $r = 0$, all firms demand capital ($\theta(0) = 0$, $L^D(0) = NB$), and the expected return to banks is negative ($\rho(0) = -E_\theta[\int_0^{B-C} F(R|\theta)dR]/B < 0$). From the first inequality in Assumption 2, it thus follows that $L^S[\rho(0)] < L^D(0)$. As $r \rightarrow r^{max}$ from below, $L^S[\rho(r)] \rightarrow L^S(\bar{R}/B - 1) > 0$ (from Theorem 1) and $L^D(r) \rightarrow 0$. So the second inequality in Assumption 2 implies $L^S[\rho(r)] > L^D(r)$ as $r \rightarrow r^{max}$ from below. Jointly the continuity requirements in Assumption 1 and the boundary conditions in Assumption 2 imply the existence of a market-clearing interest rate, r.

IV Equilibrium and credit rationing

Equipped with the results from the previous section, we are now able to give a complete characterization of the credit market equilibrium. We show that "pure" credit rationing occurs only if one assumes that lending is a one-stage process. This assumption renders the question of why firms do not bid a higher interest rate contingent on being credit-rationed meaningless, however. If, following Stiglitz and Weiss (1981, p. 398) one allows for two-stage credit allocation, demand equals supply in stage two.

To begin with, we focus on one-stage credit allocation:

ASSUMPTION 3: *Banks offer all the funds at their disposal at a single interest rate, r.*

THEOREM 2: *Suppose Assumptions 1 to 3 are satisfied. Then there exists a unique market equilibrium. The market equilibrium is characterized by either market clearing or credit rationing.*

Proof: Assumptions 1 and 2 ensure the existence of a market-clearing interest rate. Let r_m denote the lowest market-clearing interest rate ($r_m = \min\{r \,|\, L^D(r) = L^S[\rho(r)]\}$). Two cases have to be distinguished.

First, supposing $\rho(r)$ does not possess a local maximum with $\rho(r) \geq \rho(r_m)$ at an interest rate $r < r_m$, then r_m is the equilibrium interest rate. This is because supply equals demand, and there is no way to make a higher expected return with a lower interest rate.

Next, suppose to the contrary that $\rho(r)$ has one or more local maxima with $\rho(r) \geq \rho(r_m)$ at interest rates $r < r_m$ (see the lower panel of Figure 1). In this case, we can apply Theorem 5 in Stiglitz and Weiss (1981, p. 397). There is no equilibrium with market clearing, since banks can offer credit at a lower interest rate without making a lower expected return. Of the interest rates below r_m which maximize $\rho(r)$ locally, let r^* denote the one that yields the highest expected return: $r^* = \arg\max_r\{\rho(r) \,|\, r < r_m\}$ (the lowest if the maximum is not unique). From the fact that $L^D(r)$ and $L^S[\rho(r)]$ are continuous (by Assumption 1), $L^D(0) > L^S[\rho(0)]$ (by Assumption 2), and that r^* is less than the lowest market-clearing interest rate, r_m, it follows that there is excess demand for credit at r^*: $L^D(r^*) > L^S[\rho(r^*)]$. Nonetheless, r^* is the equilibrium interest rate. The equilibrium interest rate cannot be lower than r^*. This is because excess demand for credit would be even greater, so that banks could raise the interest rate and, hence, their expected rate of return. Interest rates, r, higher than r^*, would have to yield expected returns $\rho(r) > \rho(r^*)$ (according to Theorem 1, such interest rates exist). By construction, this requires $r > r_m$. Taken together, it follows that $L^S[\rho(r)] \geq L^S[\rho(r^*)] \geq L^S[\rho(r_m)] = L^D(r_m) \geq L^D(r)$. If at least one inequality is strict, r is not an equilibrium interest rate because there is excess supply. If none of the inequalities is strict, r clears the market, but is not an equilibrium interest rate because there is a lower market-clearing interest rate, viz. r_m. Q.E.D.

This is "pure" credit rationing in the sense of Stiglitz and Weiss (1981, p. 394-395): "among loan applicants who appear to be identical, some receive a loan and others do not, *and the rejected applicants would not receive a loan even if they offered to pay a higher interest rate*" (emphasis added). A drawback of the analysis so far is that the reason why the rejected firms do not get credit at a higher interest rate is that this is ruled out by the assumption of one-shot credit allocation (Assumption 3). So, following Stiglitz and Weiss (1981, p. 398), we proceed to check whether rejected firms still do not get credit if one considers the following two-stage credit allocation process:

ASSUMPTION 4: *Banks give credit in two stages. They collect funds by depositors. In stage one, they lend part or all of the funds at their disposal to borrowers at a given interest rate,* r_1*. In stage two, they lend the remainder at a higher interest rate,* $r_2 > r_1$*, to firms that have not received capital in stage one. Firms cannot make a commitment not to apply for funds in stage two.*

Assumption 4 is less restrictive than Assumption 3 in that it does not exclude the possibility that banks lend all of the funds at their disposal in stage one. Clearly, if the return function were hump-shaped, with an interior global maximum at the "bank-optimal" interest rate, r^*, this is what banks would do. So Stiglitz and Weiss (1981, p. 398) discuss two-stage credit allocation process in the context of a return function with several modes. This is an unnecessary restriction, since, by our Theorem 1, there exist interest rates, r, which yield returns, $\rho(r)$, in excess of $\rho(r^*)$ anyway. The following result shows that this has profound implications for the occurrence of credit rationing in equilibrium.

THEOREM 3: *Suppose Assumptions 1 and 2 are satisfied. If the equilibrium is market-clearing under Assumption 3, the same allocation is the unique equilibrium under Assumption 4. If the equilibrium is characterized by credit rationing under Assumption 3, then, generically, there is a unique equilibrium with two-stage credit allocation and demand equal to supply in stage two under Assumption 4.*

Proof: As in Theorem 2, if $\rho(r)$ does not possess a local maximum with $\rho(r) \geq \rho(r_m)$ at an interest rate $r < r_m$, then r_m is the equilibrium interest rate, all funds are lent in stage one, and supply equals demand.

Turning to the opposite case, where $\rho(r)$ has one or more local maxima with $\rho(r) \geq \rho(r_m)$ at interest rates $r < r_m$, we proceed in several steps. Let $\bar{L}$ denote the amount of funds lent in stage one. To begin with, we argue that $r_1 = r^*$ in a credit market equilibrium. To see this, suppose, to the contrary, that $r_1 \neq r^*$. If $r_1 < r^*$, then $\bar{L} = 0$, since $\bar{L} > 0$ would mean that banks could increase expected return by raising the interest rate to r^*. So the stage-two credit market behaves like the one-shot credit market under Assumption 3. From Theorem 2 and the case distinction made, $r_2 = r^* > r_1$. Thus, there is excess demand at r_1 in stage one, which induces firms to increase r_1 to r^*, contradicting the supposition $r_1 < r^*$. If, on the other hand, $r_1 > r^*$, we must have $\rho(r_1) > \rho(r^*)$, since otherwise banks could attract borrowers without a decrease in expected return with the lower interest rate r^*. However, from the definition of r^*, this implies $r_1 > r_m$. So it would be possible to underbid r_1 with the market-clearing interest rate r_m. This proves $r_1 = r^*$ and, hence, $\rho(r_1) = \rho(r^*)$.

The next step is to show that $\rho(r_2) = \rho(r^*)$. If $\rho(r_2) < \rho(r^*)$, then no lending takes place in stage two, and in stage one, the credit market behaves like the one-shot credit market under Assumption 3. According to Theorem 2 and the case distinction made, there is excess demand in stage one. This is not an equilibrium, because a bank could keep a small amount of loanable funds for stage two and, according to Theorem 1, make expected return $\rho(r^{max}) = \bar{R}/B - 1$ by lending to rejected borrowers at the interest rate $r_2 = r^{max}$. If, on the other hand, $\rho(r_2) > \rho(r^*)$, then no lending takes place in stage one, and the credit market in stage two behaves like the one-shot credit market, so that $\rho(r_2) = \rho(r^*)$, a contradiction. This proves $\rho(r_2) = \rho(r^*)$ $(= \rho(r_1))$. Loan supply is thus $L^S[\rho(r^*)]$.

We proceed to demonstrate that $L^S[\rho(r^*)] > \bar{L}$. That is, some lending activity takes place in stage two. This is an important step in the proof, as it implies that the equilibrium allocation under Assumption 3 is not an equilibrium here. Supposing that $L^S[\rho(r^*)] = \bar{L}$, the one-shot credit rationing equilibrium arises in stage one. As explained above, Theorem 1 implies that a bank can make expected return $\rho(r^{max}) = \bar{R}/B - 1$ by lending a small amount of funds to rejected borrowers at $r_2 = r^{max}$ in stage two. So $L^S[\rho(r^*)] = \bar{L}$ is inconsistent with credit market equilibrium.

We are now in a position to prove that a credit market equilibrium with $r_1 = r^*$, $\rho(r_2) = \rho(r^*)$, $L^S[\rho(r^*)] > \bar{L}$, and market clearing in stage two exists. As shown in the proof of Theorem 2, there is excess demand at $r_1 = r^*$: $L^D(r^*) > L^S[\rho(r^*)]$. Together with the finding that $L^S[\rho(r^*)] > \bar{L}$, it follows that a positive fraction, $1 - \bar{L}/L^D(r^*) > 0$, of the firms that demand capital do not receive funds in stage one. Because of the non-observability of borrower types, these firms are chosen randomly from the pool of credit applicants. Demand for credit in stage two is thus $[1 - \bar{L}/L^D(r^*)]L^D(r_2)$, and the relative frequencies of borrower types in stage two are the same as in stage one, so that $\rho(r)$ continues to give the expected return on lending. Competition enforces that $r_2 = r^{**}$, where r^{**} is the lowest interest rate, r_2, greater than r^* which satisfies $\rho(r_2) = \rho(r^*)$. Theorem 1 and the continuity of the return function, $\rho(r)$, implied by Assumption 1, ensure the existence of a solution. This is the crucial property of the return function exploited by Stiglitz and Weiss (1981, p. 398) in the proof of their Theorem 6, which we adapt now to our purposes. By construction, $r^{**} \geq r_m$. Equality of loan demand and supply in stage two requires $[1 - \bar{L}/L^D(r^*)]L^D(r^{**}) = L^S[\rho(r^*)] - \bar{L}$. Let $\bar{L}^*$ denote the solution to this equation:

$$\bar{L}^* = \frac{L^S[\rho(r^*)] - L^D(r^{**})}{L^D(r^*) - L^D(r^{**})} L^D(r^*).$$

From $L^D(r^*) > L^S[\rho(r^*)]$ and $r^{**} \geq r_m$, we have $L^D(r^*) > L^S[\rho(r^*)] \geq L^S[\rho(r_m)] = L^D(r_m) \geq L^D(r^{**})$. This implies that the numerator is non-negative and the denominator is positive, so that $\bar{L}^* > 0$. Furthermore, $L^D(r^*) > L^S[\rho(r^*)]$ implies $\bar{L}^* < L^S[\rho(r^*)]$. Banks cannot raise expected return on lending in either stage one or stage two. This is because, by definition, $\rho(r_1) < \rho(r^*)$ for all $r_1 < r^*$, and $\rho(r_2) < \rho(r^*)$ for $r^* < r_2 < r^{**}$. So in order to raise expected return, banks would have to set interest rates $r_1 > r^*$ or $r_2 > r^{**}$, respectively, which would not attract firms. They are indifferent between lending in stage one or in stage two. So a credit market equilibrium prevails.

To prove generic uniqueness, it remains to show that given $r_1 = r^*$, $r_2 = r^{**}$, and $L^S[\rho(r^*)] > \bar{L}$, generically, stage-one lending levels $\bar{L} \neq \bar{L}^*$ are not consistent with an equilibrium. To see this, notice that for a given decrease in $\bar{L}$, stage-two loan demand, $[1 - \bar{L}/L^D(r^*)]L^D(r^{**})$, increases less than one-to-one (since $L^D(r^*) > L^D(r^{**})$), while stage-two loan supply, $L^S[\rho(r^*)] - \bar{L}$, rises exactly one-to-one. So if $0 < \bar{L} < \bar{L}^*$, there is excess supply in stage two. This is not consistent with an equilibrium in the credit market, since competition would drive down r_2 and, hence, expected return. If, on the other hand, $\bar{L}^* < \bar{L} < L^S[\rho(r^*)]$, there is excess demand at r^{**}. If r^{**} (i.e., the lowest interest rate greater than r^* which yields expected return $\rho(r^*)$) does not coincidentally maximize $\rho(r)$ locally, banks can raise expected return with a higher interest rate, r_2. This proves that, generically, $r_1 = r^*$, $r_2 = r^{**}$, and $\bar{L} = \bar{L}^*$ represent the unique credit market equilibrium. Q.E.D.

Theorem 3 makes it clear that paying attention to the fact that the return function cannot be hump-shaped has a profound impact on the equilibrium analysis. In order to be able to check whether rejected applicants would possibly receive a loan at a higher interest rate, r_2, we have to replace Assumption 3 with the less restrictive Assumption 4. Whether or not equilibrium "credit rationing" can occur becomes a semantic issue, then. If there is an excess demand for credit in stage one of the credit allocation process, rejected applicants do receive a loan at the higher interest rate r^{**} in stage two (except possible in the non-generic case that $\rho(r^{**}$ is a local maximum of $\rho(r)$; see below). So the above-mentioned definition of pure credit rationing put forward by Stiglitz and Weiss (1981, p. 394-395) is not satisfied. Nonetheless, one could speak of credit rationing in stage one. Three concluding remarks are in order. First, in order to substantiate the claim that credit rationing à la Stiglitz and Weiss (1981, p. 394-395) cannot occur, it is essential that we give an exhaustive characterization of the credit market equilibrium, rather than confine attention to examples. Second, the fact that firms cannot make a commitment not to apply for funds in stage two (Assumption 4) is crucial. To see this, notice that only "risky" firms, with $\theta \geq \theta(r^{**})$, demand funds in stage two. Since $L^S[\rho(r^*)] > \bar{L}^*$, firms with $\theta < \theta(r^{**})$ face a lower probability of getting finance than in the one-shot credit market and, therefore, are in a worse position. Assumption 4 rules out the possibility that they signal that they are "safe" by making a commitment not to apply for a loan at r_2 in stage two. Third, turning to the non-generic case that $\rho(r^{**})$ is a local maximum of the return function, let r' denote the market-clearing interest rate in stage two, given $\bar{L}$: $[1 - \bar{L}/L^D(r^*)]L^D(r') = L^S[\rho(r^*)] - \bar{L}$. If $\rho(r) > \rho(r^*)$ for some r in the interval $r^{**} < r \leq r'$, banks can raise expected return with a higher interest rate, which is inconsistent with credit market equilibrium. If this condition is not satisfied, then $r_1 = r^*$, $r_2 = r^{**}$, and $\bar{L}$ represent a credit market equilibrium with credit rationing in stage two (this is essentially the situation described by Theorem 2 in the case of a flat loan supply function, $L^S(\rho)$).

V Redlining

In Section IV of their seminal article, Stiglitz and Weiss (1981, p. 406) introduce observationally distinguishable classes of borrowers to their model. The idea is elaborated upon by John Riley (1987) and Stiglitz and Weiss (1987). This extension of the model allows it to distinguish ("type-a") credit rationing and "redlining" ("type-b credit rationing"). The latter is said to occur when an entire class of borrowers is denied credit (see Stiglitz and Weiss, 1981, p. 395, 1987, p. 228). Both Stiglitz and

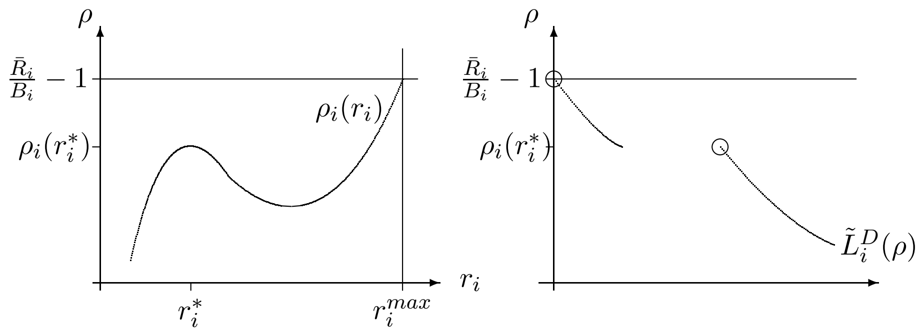

Figure 2: Class i's loan demand

Weiss (1981, p. 406) and Riley (1987, p. 225) assume explicitly that for each borrower class, the return function is hump-shaped, such that there is an interior bank-optimal interest rate. They argue that redlining is an important property of the extended model (see Stiglitz and Weiss, 1981, p. 407, 1987, p. 230, and Riley, 1987, p. 226). In this section, we show that given the true shape of the return function described in Theorem 1, borrower classes are redlined only if at the equilibrium expected rate of return, they would not receive credit with perfect information either.

Suppose there is a discrete number, n (≥ 1), of borrower classes. We use the subscript i $(\in \{1, 2, \ldots, n\})$ to distinguish variables referring to different classes. Each class of borrowers is structured as described in Section I. Banks can observe to which class, i, a borrower belongs. They know the distribution of borrower types, $G_i(\theta_i)$, within this class. But they cannot observe the borrower's risk group, θ_i. In Assumption 2, it suffices if the condition $L^S(\bar{R}_i/B_i - 1) > 0$ is satisfied for one borrower class, i.

Clearly, in a perfect capital market, banks would not finance projects with an expected internal rate of return, $\bar{R}_i/B_i - 1$, that falls short of the rate of return paid to their depositors, ρ. The following result shows that the same condition rules out redlining here:

THEOREM 4: *Class i is redlined only if $\bar{R}_i/B_i - 1 < \rho$.*

Proof: Since banks can observe to which class, i, a borrower belongs, they can set class-specific interest rates, r_i. For each class, i, banks calculate the return function, $\rho_i(r_i)$. Competition among banks enforces that any expected rate of return, ρ, is obtained by means of the lowest possible interest rate, r_i. Let $r_i(\rho)$ denote the function that assigns this competitive interest rate to any feasible expected rate of return, ρ:

$$r_i(\rho) = \min \left\{ r_i \,\middle|\, \rho(r_i) = \rho,\, 0 \leq \rho \leq \frac{\bar{R}_i}{B_i} - 1 \right\}.$$

The crucial observation is that, by virtue of Theorem 1, $r_i(\rho)$ is well-defined for rates of return, ρ, up to the projects' expected internal rate of return, $\bar{R}_i/B_i - 1$, as $\bar{R}_i/B_i - 1$ can be achieved with

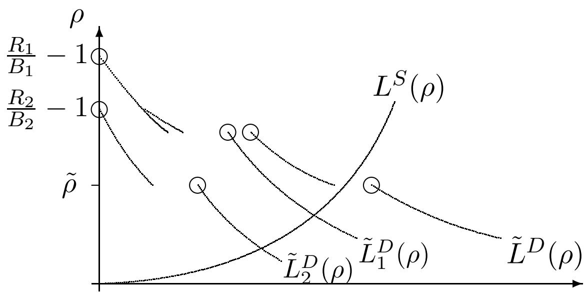

Figure 3: Credit market equilibrium with distinguishable borrower classes

$r_i = r_i^{max}$. Q.E.D.

Stiglitz and Weiss (1981, p. 407) stipulate that firms with high expected returns might be redlined if they are especially risky, even though less profitable firms are not. This is not consistent with Theorem 4. We proceed to analyze the credit market equilibrium. In doing so, we follow Riley (1987). We investigate the conditions for credit rationing and redlining in a credit market equilibrium (i.e., with ρ determined endogenously).

As ρ increases, starting from $\rho = 0$, the function $r_i(\rho)$ rises continuously until $r = r_i^{max}$ or a local maximum of the return function, $\rho_i(r')$ say, is reached. As ρ rises above $\rho_i(r')$, the function $r_i(\rho)$ jumps discontinuously to the level $r'' = \arg\min_r\{\rho_i(r) > \rho_i(r')\}$. If there is another local maximum, $\rho_i(r''')$, with $\rho_i(r''') > \rho_i(r'')$ at an interest rate $r''' > r''$, there is another discontinuity; and so on. Thus, $r_i(\rho)$ rises monotonically, with discontinuities at those rates of return, ρ, that represent local maxima of the return function, $\rho_i(r)$, which exceed the values of local maxima of the return function at lower interest rates. Class i's loan demand as a function of ρ is

$$\tilde{L}_i^D(\rho) = \begin{cases} L_i^D[r_i(\rho)]; & \text{for } \rho \leq \frac{\bar{R}_i}{B_i} - 1 \\ 0; & \text{for } \rho > \frac{\bar{R}_i}{B_i} - 1 \end{cases}$$

(see Figure 2). Notice the following two properties of $\tilde{L}_i^D(\rho)$. First, the function is discontinuous at those rates of return, ρ, where $r_i(\rho)$, displays a discontinuity. Second, the function is continuous at $\bar{R}_i/B_i - 1$. To see this, recall from Theorem 1 that $\rho = \bar{R}_i/B_i - 1$ is the unique maximum of $\rho_i(r_i)$ and is reached by the maximum interest rate, r_i^{max}, at which the riskiest group, θ_i^{max}, ceases to demand capital. Consequently, $r_i(\rho) \to r_i^{max}$ and $\tilde{L}_i^D[r_i(\rho)] \to N_i B_i[1 - G_i(\theta_i^{max})] = 0$ as $\rho \to \bar{R}_i/B_i - 1$ from below. Aggregate loan demand is $\tilde{L}^D(\rho) = \sum_{i=1}^n \tilde{L}_i^D(\rho)$. $\tilde{L}^D(\rho)$ is monotonically decreasing, with $\tilde{L}^D(\rho) = 0$ for $\rho \geq \max_i\{\bar{R}_i/B_i - 1 \,|\, i \in \{1, 2, \ldots, n\}\}$ and with discontinuities where any of the $\tilde{L}_i^D(\rho)$ functions is discontinuous (an example with $n = 2$ is illustrated in Figure 3). Loan supply, $L^S(\rho)$ increases with ρ. Two cases have to be distinguished.

In the first case, the demand and supply curves intersect:

THEOREM 5: *Suppose Assumptions 1 and 2 are satisfied and there is a $\tilde{\rho}$ such that $\tilde{L}^D(\tilde{\rho}) = L^S(\tilde{\rho})$. Then there exists a unique market equilibrium. $\tilde{\rho}$ is the equilibrium interest rate, and the market equilibrium is characterized by market clearing.*

Otherwise, there exists a $\tilde{\rho}$ such that $\tilde{L}^D(\tilde{\rho}) > L^S(\tilde{\rho})$ and $\tilde{L}^D(\rho) < L^S(\rho)$ for $\rho > \tilde{\rho}$ (i.e., the demand curve "jumps" below the supply curve, as in Figure 3). Consider the borrower class, i, whose loan demand function, $\tilde{L}_i^D(\rho)$ is discontinuous at $\tilde{\rho}$ (if more than one demand function is discontinuous at $\tilde{\rho}$, the set of classes). By the same reasoning as in the proof of Theorem 2, if Assumption 3 holds true, then $\tilde{\rho}$ is the equilibrium interest rate and there is credit rationing in that class, i. By the same reasoning as in the proof of Theorem 3, if Assumption 4 holds true, banks offer loans at the interest rates r_i^* and r_i^{**} and make loans worth $\bar{L}_i^*$ in period 1, where r_i^{**} is the lowest interest rate greater than r_i^* with $\rho_i(r_i^{**}) = \rho_i(r_i^*)$ and

$$\bar{L}_i^* = \frac{L^S(\tilde{\rho}) - \sum_{j=1, j\neq i}^{n} L_j^D(\tilde{\rho}) - L_i^D(r^{**})}{L_i^D(r^*) - L_i^D(r^{**})} L_i^D(r^*).$$

This proves:

THEOREM 6: *Suppose Assumptions 1 and 2 are satisfied and there is not a $\tilde{\rho}$ such that $\tilde{L}^D(\tilde{\rho}) = L^S(\tilde{\rho})$. Then there exists a unique market equilibrium. Under Assumption 3, $\tilde{\rho}$ is the equilibrium interest rate, and the market equilibrium is characterized by credit rationing. Under Assumption 4, credit is allocated in two steps, and demand is equal to supply in stage two.*

The fact that, given Assumption 3, generically, one class, i, at most is credit-rationed is emphasized by Riley (1987, p. 226). Redlining in equilibrium can now be ruled out quite generally:

THEOREM 7: *Suppose $\tilde{\rho} < \bar{R}_i/B_i - 1$ for all $i \in \{1, 2, \ldots, n\}$. Then no borrower class is redlined in equilibrium.*

Proof: This follows immediately from Theorems 4-6. Q.E.D.

VI Conclusion

Contrary to what is consistently assumed in the literature, the return function cannot be hump-shaped in the Stiglitz-Weiss (1981) model. This is not just a matter of expositional convenience. With a single class of borrowers, banks offer credit in two stages. Demand possibly exceeds supply in stage one, but not in stage two. With several observationally distinguishable borrower classes, a borrower class is redlined only if the expected internal rate of return of their projects falls short of the rate required by the banks' depositors, in which case they would not receive credit in a perfect capital market either. Due to the misguided focus on a hump-shaped return function, these properties of the credit market equilibrium have been going unnoticed ever since the publication of Stiglitz and Weiss' (1981) seminal work.

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